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QUARTERLY PROGRESS REPORT ON CONTRACT NAS 8-2604 FOR MEASUREMENTS AND IMPROVEMENTS OF TWO HYDROGEN MASERS

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CONTENTS

1. INTRODUCTION

The work reported herein covers the period March 1 - June 30 and is concerned chiefly with improvements to the maser and the task of making short term measurements. This will be the last of the Quarterly Reports; henceforth the reporting interval will be one month.

2. STATUS OF IMPROVEMENTS

a) Solid Dielectric Cavity

A mounting and tuning mechanism has been built for incorporating the solid cavity in either the NASA masers or the Varian experimental maser.

b) NASA Masers

NASA maser No. 2 is completely under the new thermal control and operating satisfactorily.

3. MEASUREMENT SYSTEM FOR INTERCOMPARISON WITH OTHER MASERS

A system for phase locking a crystal oscillator to a hydrogen maser for making long term measurements via VLF and Loran "C" is under construction.

a) Short Term Measuring Apparatus

The cross correlation system has been completed and is in operation. Attached is a copy of a draft for a paper on this device.

The results are as expected. The r.m.s. frequency deviation over short time intervals τ is given by a $1/\tau^{\frac{3}{2}}$ law indicating the dependence or additive thermal noise kT. For τ greater than 10 seconds the behavior should be that predicted by a $1/\tau^{\frac{1}{2}}$ law.

Measurements and conclusions will be reported next month.

A series of Maser Seminars for NASA scientists and technicians will be held at Varian's Q.E.D. Activity in Beverly, Mass., July 13-18. Visitors for the Navy and the Air Force have been invited to participate. A set of notes relating to theory as well as hardware is in preparation. In general, the work will be divided equally between the lectures and the laboratory.

A CROSS-CORRELATION TECHNIQUE FOR MEASURING THE SHORT TERM PROPERTIES OF STABLE OSCILLATORS

By making the product of two signals, information can be obtained of the magnitude of the r.m.s. amplitude and phase variations as well as any correlation that may exist between them. The apparatus referred to here consists of a means for transforming the signals to a lower frequency, 600 cps from 1.4 kmc so that the signals can be compared in an analog manner by an electronic multiplier. Figure 1 shows the system. Both channels use a common pump oscillator for the parametric amplifier as well as common local oscillators for conversion to 30 mc and subsequently to 600 cps. The signals at each input to the multiplier are assumed to be of the form

$$V_i(t) = \left[x_i(t) + A \right] \cos \omega_i t - y_i(t) \sin \omega_i t$$

where $i = 1,2$ and $\omega_1 - \omega_2 = \Omega$, a slow angular frequency of about 1 radian per minute.

$y_i(t)$ and $x_i(t)$ are instantaneous variations of voltage and are defined as consisting of two components:

$$x_i(t) = w_i(t) + z_i(t) \qquad \overline{w_i z_i} = 0 \qquad \overline{w_i v_i} = 0$$

$$y_i(t) = u_i(t) + v_i(t) \qquad \overline{u_i v_i} = 0 \qquad \overline{u_i z_i} = 0$$

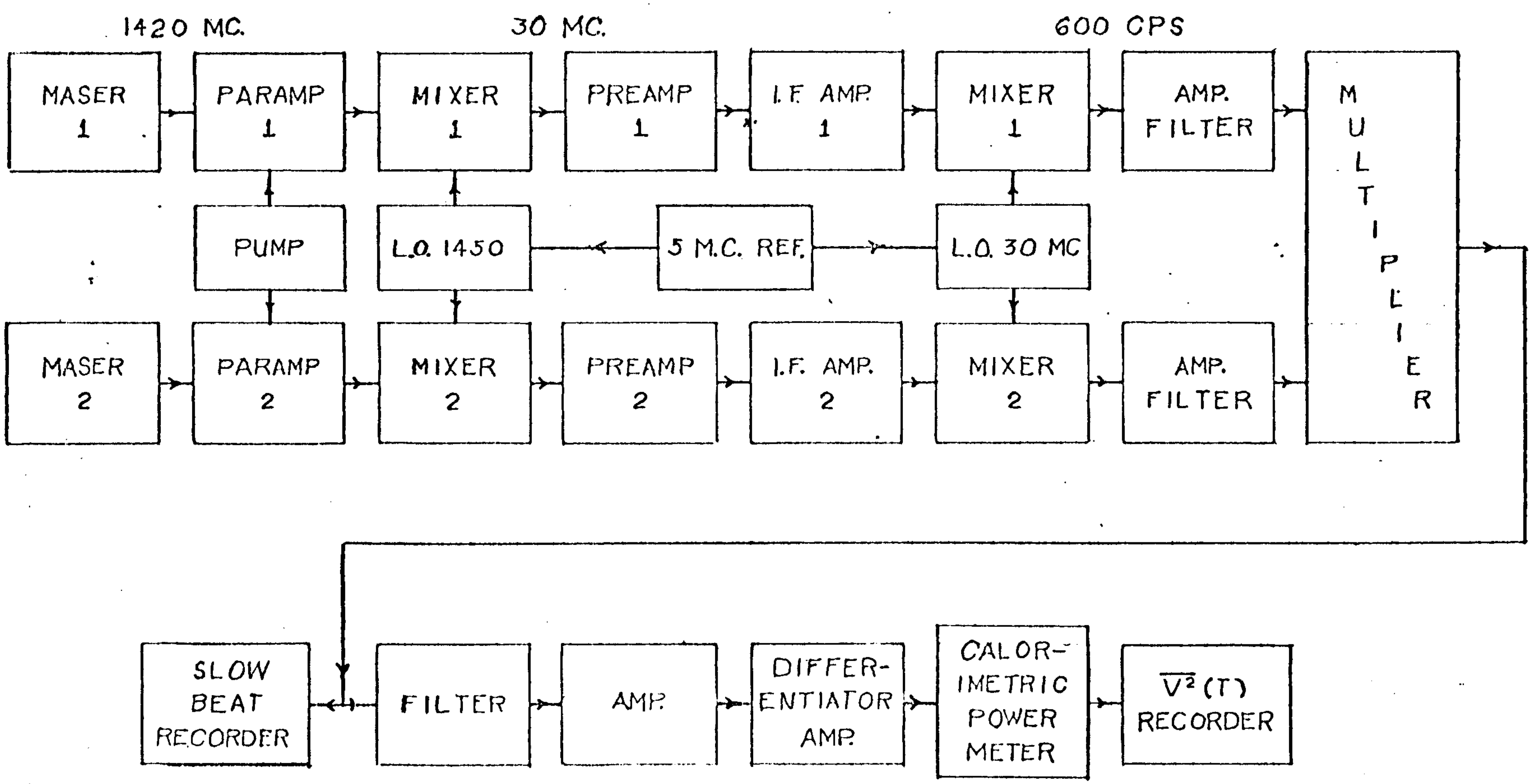
where $w_i(t)$ is the in-phase component of oscillator noise,
$z_i(t)$ is the in-phase component of receiver noise,
$u_i(t)$ is the quadrature component of oscillator noise,
$v_i(t)$ is the quadrature component of receiver noise.

In general, $v_1^2(t) = z_1^2(t)$ and $v_2^2(t) = z_2^2(t)$.

Let $E(t)$ be the voltage output from the multiplier.

$$\begin{aligned} E(t) &= K V_1 V_2 \\ &= K r_1 r_2 \cos(\omega t + \phi_1) \cos(\omega t + \phi_2 + \Omega t) \\ &\doteq K \frac{r_1 r_2}{2} \left[\cos \Omega t + (\phi_1 - \phi_2) \sin \Omega t \right] \end{aligned}$$

The approximation here is that terms involving $\left(1 - \frac{\phi^2}{2}\right)$ have been made equal to 1.



SHORT TERM NOISE MEASUREMENT

The signal is seen to involve a slow beat at $\cos \Omega t$. The values of the fluctuations due to terms in x and y about the d.c. level of the slow beat are of interest. If the beat is "stopped" so that $\Omega t = 0$,

$$E_o(t) = K \frac{r_1 r_2}{2}$$

$$= K \frac{A^2}{2}\left[1 + \frac{x_1}{A}\right]\left[1 + \frac{x_2}{A}\right] \qquad \text{if } A_1 = A_2$$

$$= \frac{KA^2}{2} + \frac{KA}{2}(x_1 + x_2) \qquad \text{since } x_i \ll A$$

--the first being the d.c. term, the second the term involving fluctuation in signal amplitude.

Stopping the beat at $\Omega t = \frac{\pi}{2}$,

$$E_{\frac{o}{2}}(t) = K \frac{r_1 r_2}{2}(\phi_1 - \phi_2)$$

$$= \frac{KA^2}{2}\left[1 + \frac{x_2}{A} + \frac{x_1}{A}\right]\left[\phi_1 - \phi_2\right] \qquad \phi_i \approx \frac{x_i}{A}$$

$$= \frac{KA^2}{2}(\phi_1 - \phi_2)$$

$$= \frac{KA}{2}(y_2 - y_1)$$

At this point the signal can be used to determine the magnitudes of $\overline{x^2(t)}$ and $\overline{y^2(t)}$ by passing the signals to a square law device such as a power meter where $\frac{E^2}{R}$ is obtained in terms of a reading D.

$$D = \frac{E^2}{R} = \frac{K^2A^2}{R4}(y_2^2 + y_1^2 - 2y_1y_2)$$

Due to this xy correlation, phase fluctuations common to both channels due to the common local oscillators and amplifier pump are eliminated.

$$\Delta\phi_T^2 = \frac{4D_{\pi/2}R}{K^2A^4}$$ and consists of the component of phase variations between the two signals.

Amplitude fluctuations given by

$$\Delta x_T^2 = \frac{4D_0R}{K^2A^2}$$ consist of both components of amplitude variation.

If the signals contain nothing else but random or Gaussian noise, then there is no further information available and a calibration of $\overline{x_T^2}$ can be obtained by using a noise generator. The noise generator produces $x(t)$ and $y(t)$ fluctuations that are uncorrelated and

$$D_{o_n} = \frac{E^2}{R} = \frac{K^2 A^2}{4R} \left(\overline{z_1^2} + \overline{z_{1n}^2}\right) = \frac{K^2 A^2}{4R} \left(\overline{v_1^2} + \overline{v_{1n}^2}\right)$$

where v_{1n} and z_{1n} is the representation of excess noise on one channel.

Normally the noise generator will be used in place of a signal input, and under these conditions

$$D_{o_n} = \frac{E^2}{R} = \frac{K^2 A^2}{4} \left[\frac{w_1^2 + z_1^2}{R} + k(T - T_0) B \right]$$ will be the

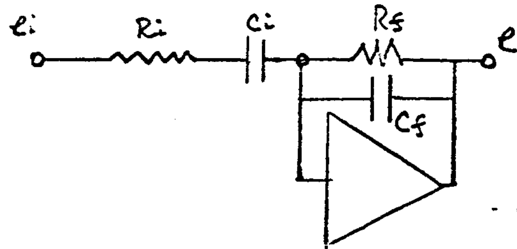
total deflection. If we write the effective noise voltage squared the input, due to the available noise power FRT_oB (where F is the noise figure of the receiver) as $V_1^2 = kTRB$ and consider for the moment that $w_1^2 = 0$ or is very small (according to the theory of the H masers), then

$$D_{o_n} = \frac{K^2 A^2}{4} kTB \; (T_0 F + T - T_0)$$

The factor $Y = \dfrac{T - T_0 + F T_0}{F T_0} = 1 + \dfrac{1}{F}\left[\dfrac{T}{T_0} - 1 \right] = \dfrac{D_{o_n}}{D_0}$ is measurable

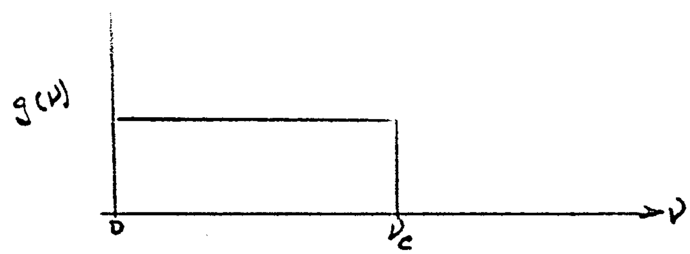
by turning the terminated noise source on and off. From this value one may determine F. Care must be taken in considering the bandwidth B when using a broad band noise source since the calibrating noise will be introduced through the idler channel of the parametric amplifier or through the image frequency of the superhetrodyne receiver converter.

The properties of the signal that we seek are those of frequency and not of phase. To observe these, the phase signal must be processed further to obtain the rate of change of phase, $\dot{\phi}$, measured over an interval of time, τ. One may obtain $\dot{\phi}$ by electronic differentiation using an operational amplifier and a rc network.



$$e = \frac{-j\omega R_f C_i e_i}{(1+j\omega R_f C_f)(1+j\omega R_i C_i)}$$
$$\doteq -j\omega R_f C_i e_i$$

By passing the signal through a low pass filter where the cutoff frequency ν_c is related to τ by $\tau = \frac{1}{2\nu_c}$, one obtains $\dot{\phi}_\tau$.



The process of specifying the time interval τ makes use of the sampling theorem:

> If a function G(t) contains no frequencies higher than W cycles per second it is completely determined by giving its ordinates at a series of points $\frac{1}{2W}$ apart extending throughout the time domain.

This function, given by a series of points $\frac{1}{2\nu_c}$ apart, is described by the signal from the filter, and whether the filter precedes or follows the differentiator has no significance except when considerations are made of power levels and impedances of the equipment in use.

The signal after low pass filtering and differentiation and when the slow beat, Ωt, is at $\frac{\pi}{2}$, $\frac{3\pi}{2}$ etc., is proportional to $\overline{\dot{\phi}}_\tau$, where $\overline{\dot{\phi}}_\tau$ is the time rate of change of phase taken over intervals τ seconds. This quantity varies with time and it is the quantity $(\overline{\dot{\phi}_\tau})_{avg}$ that is of interest. This stationary value is obtained by putting the signal into a square law device that has a long time constant such as a calorimetric power meter. The output from this square law device is led to one channel of stripchart recorder. Another channel of the recorder is used to record the slow beat, $\cos \Omega t$, and the correlation data can read at various positions of Ωt, thus identifying the phase and amplitude variations.

The signal into the differentiator from the cross multiplier and filter is given by

$$E_{\pi/2}(\tau) = \frac{KA}{2}\left[y_2(\tau) - y_1(\tau)\right] = \frac{KA^2}{2}\left[\phi_2(\tau) - \phi_1(\tau)\right] .$$

where the action of the filter is designated by the change from the symbol t to τ.

By squaring, differentiating, and averaging this value, one obtains

$$(-R_0C_0)^2\, \overline{\dot{E}^{\,2}_{\pi/2}(\tau)} = (-R_0C_0)^2\, \frac{K^2A^2}{4}\left[\overline{\dot{y}_2^{\,2}}(\tau) + \overline{\dot{y}_1^{\,2}}(\tau)\right]$$

and similarly with x instead of y for $\overline{\dot{E}_0^2}(\tau)$.

The effect of thermal noise and receiver excess noise is of interest here as the frequency response of the differentiator is proportional to frequency.

$$g(\nu) = 2\pi R_0 C_0 \nu$$

The square of this response will be the effect on the power vs. frequency response, and, when driven from a source of power having spectral density $kT\,d\nu$, the over-all power as a function of the filter cutoff frequency ν_c is

$$\text{Noise Power} = \int_{\nu}^{\nu_c} g(\nu)^2 kT d\nu = \frac{4\pi^2 R_0^2 C_0^2 kT\, \nu_c^3}{3}$$

The output from the differentiator, which is given by

$$D_0 = K\frac{(R_0C_0)^2}{R}\, \overline{\dot{E}_0^2}(\tau)$$

$$= \frac{R_0^2C_0^2}{R}\,\frac{K^2A^2}{4}\left[\overline{\dot{x}_1^2} + \overline{\dot{x}_2^2}\right]$$

can now be related to thermal noise in the following way:

$$= \frac{R_0^2C_0^2}{R}\,\frac{K^2A^2}{4}\left[\overline{\dot{w}_1^2} + \overline{\dot{w}_2^2} + \overline{\dot{z}_1^2} + \overline{\dot{z}_2^2}\right]$$

Remembering that w_1 and w_2 are both small compared to z, then

$$= \frac{R_0^2C_0^2K^2A^2}{4R}\left[\overline{\dot{z}_1^2} + \overline{\dot{z}_2^2}\right]$$

$$= \frac{R_0^2 C_0^2 K^2 A^2}{4R} \left[\frac{4\pi^2}{3} kTR\,(F_1 + F_2)\,\nu_c^3 \right]$$

where F_1 and F_2 are the noise figures of channel 1 and 2, respectively. F_1 and F_2 can be obtained using a noise source as before. The power vs. frequency response of the system behaves as ν^2; the effective integrated bandwidth is proportional to ν^3 and is a constant of the measurement. By making the measurement of F_1 and F_2 it is possible to use the noise of the system as a calibration for the phase fluctuations given by $\dot{\nu}^2$.

$$Q_{\pi/2}^2 = \frac{R_0^2 C_0^2 K^2 A^2}{4R} \left[\frac{4\pi^2}{3} kTR\,(F_1 + F_2)\,\nu_c^3 + \overline{\dot{u}_1^2} + \overline{\dot{u}_2^2} \right]$$

The r.m.s. fractional frequency deviation of measurements made over a time interval τ is given by

$$\frac{\langle \Delta f_b^2 \rangle_\tau^{\frac{1}{2}}}{f} = \frac{1}{2\pi f} \frac{1}{P^{\frac{1}{2}}} \left\langle \frac{\overline{\dot{u}^2(\tau)}}{R} + \frac{\overline{\dot{u}_2^2(\tau)}}{R} + \frac{4\pi^2 kT\,2F}{24\,\tau^3} \right\rangle^{\frac{1}{2}}$$

where $P = \frac{A^2}{R}$ and if $F_1 = F_2 = F$,

where Δf_b is the result of beating two masers. For a single maser the result must be multiplied by $\frac{1}{\sqrt{2}}$.

It is seen here that in the absence of frequency fluctuations due to the oscillator, the short term stability will depend on $\frac{1}{T^{3/2}}$. The question of how to account for thermal noise is best considered by considering the oscillator to be at some temperature T_1. Accompanying the signal from the device will be a noise power kT_1 per unit bandwidth. If the signal is led to a receiver having effective noise temperature T_1 (or at temperature T_1 with noise figure $F = 1$), the system is in thermodynamic equilibrium and it makes little sense to consider the noise as wholly from one or the other source. In general the noise temperature of the hottest element will determine the limiting frequency stability.

The effect of noise on frequency is given by

$$\frac{\langle \Delta f_n^2 \rangle^{\frac{1}{2}}}{f} \Bigg|_\tau = \frac{1}{2\pi f} \frac{1}{P^{\frac{1}{2}}} \left\langle \frac{4\pi^2 kTF}{24\,\tau^3} \right\rangle^{\frac{1}{2}} \qquad \text{for one channel.}$$

The effect of noise on the radiating atoms is given by

$$\frac{\langle \Delta f_m{}^2 \rangle^{\frac{1}{2}}}{f} = \frac{.113\gamma^2}{2\pi f}\left\langle \frac{kT}{P\tau} \right\rangle^{\frac{1}{2}} \qquad \text{valid for } \tau > \frac{1}{\gamma}$$

If the term $\frac{1}{2\pi f}\left\langle \frac{u^2(t)}{RP} \right\rangle^{\frac{1}{2}}$ is of this form, then we can combine them to give

$$\frac{\langle \Delta f_{total} \rangle^{\frac{1}{2}}}{f} = \left\langle \frac{\Delta f_n{}^2}{f} + \frac{\Delta f_m{}^2}{f} \right\rangle^{\frac{1}{2}}$$

$$\frac{\langle \Delta f_{tot} \rangle^{\frac{1}{2}}}{f} = \frac{1}{2\pi f}\left\langle \frac{kT}{P}\left[\frac{(.226)^2\gamma^2}{\tau} + \frac{\pi^2}{6}\frac{F}{\tau^3} \right] \right\rangle^{\frac{1}{2}} .$$

At what τ will the contributions from the frequency fluctuations due to thermal noise be equal to those induced by noise upon the maser oscillation?

$$\frac{(.226)^2\gamma^2}{\tau_0} = \frac{\pi^2}{6}\frac{F}{\tau_0^3}$$

$$\tau_0 = \frac{\pi}{\gamma(.226)}\sqrt{\frac{F}{6}} \qquad \text{for} \quad F = 7.3$$

$$\gamma = 3.3 \text{ sec.}^{-1}$$

$$\tau_0 = 4.65 \text{ sec.}$$

For time intervals shorter than τ_0 the dependence of $\frac{\langle \Delta f^2 \rangle^{\frac{1}{2}}}{f}$ on τ will approach $\frac{1}{\tau^{3/2}}$. For time intervals larger than τ_0 the behavior should approach $\frac{1}{\tau^{1/2}}$. The effect of variations in frequency due to causes other than noise will very likely hide this behavior.